The Dirichlet Model: Analysis of a Market and Comparison of Estimation Procedures.

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This paper examines the Dirichlet model describing consumer behaviour. The model estimates brand performance measures in the case of repeat purchases over a set of brands. The Dirichlet model relies on some assumptions such as stationarity and the fact that the market is unsegmented. Its formulation derives from a combination of the Negative Binomial and the Dirichlet distributions. Various estimation methods have been proposed. The original one is an iterative procedure based on the method of moments and requires as inputs only aggregated quantities, such as brand penetrations and average purchase rates. There is also an estimation method based on likelihood maximization which requires raw individual or household panel data. The method of moments deserves attention, since raw panel data are frequently not available to researchers and/or enterprises. In this paper, the Dirichlet model is used to analyze the Italian beer market as a by-product of the main objective, which is to compare two estimation procedures available on-line for the method of moments: one based on an Excel Workbook and the other written in R. Neither procedures are very robust in the presence of atypical brands in the market.

Keywords: Dirichlet model, consumer behaviour, estimation, market segmentation

Introduction

The Dirichlet model describes how frequently-bought branded consumer products are purchased when the market is stationary and unsegmented. It was developed by Goodhardt, Ehrenberg and Chatfield (1984) and in the following years was shown to be applicable to many product categories and to have substantial uses, particularly with regard to analysis of brand performance measures.

In this paper, the Dirichlet model is applied not only to describe the Italian beer market but, more importantly, to compare results obtained estimating model parameters with two software packages available on-line: an Excel-based one, written by Kearns (2002) and that developed by Chen (2008) using programming language R.

The Dirichlet model

The Dirichlet model describes patterns of repeat purchases of brands within a product category. It models simultaneously the counts of the number of purchases of each brand over a period of time, so that it describes purchase frequency and brand choice at the same time. It assumes that consumers have an experience of the product category, so that they are not influenced by previous purchase and marketing strategies; for this reason, consumer characteristics and marketing-mix instruments are not included in the model. As the market is assumed to be stationary, these effects are already incorporated in each brand market share which influences other brand performance indexes calculated by the model. The market is also assumed to be unsegmented. The theory and development of the model is fully described in Ehrenberg (1972).

Let us consider a sample of n consumers making purchases in a market with g brands. The specification of the Dirichlet model derives from the following assumptions:

1) The number of purchases of each brand j, with $j=1,\dots,g$, $r_1,\dots, r_g$, made by the i-th consumer over a succession of purchases, can be modelled by a multinomial distribution with parameters $r, p_1,\dots, p_g$:

$$P(r_1,\dots,r_g) = r!\prod_{j=1}^{g}\left(\frac{p_j^{r_j}}{r_j!}\right)$$

where r is the total number of purchases in the product category.

2) The probabilities p_j vary among individuals according to a Dirichlet distribution with parameters $\alpha_1,\dots,\alpha_g$:

$$f(p_1,\dots,p_{g-1}\mid\alpha_1,\dots,\alpha_g) = \frac{\Gamma(\alpha_1+\dots+\alpha_g)}{\Gamma(\alpha_1)\dots\Gamma(\alpha_g)}\, p_1^{\alpha_1-1}\dots p_{g-1}^{\alpha_{g-1}-1}(1-p_1-\dots-p_{g-1})^{\alpha_{g-1}}$$

3) Successive purchases by the ith consumer are independent. The number of purchases n_i made by the ith consumer in each of a succession of equal non-overlapping periods of length T, follows a Poisson distribution with mean $\mu_i T$.

4) Mean purchasing rates vary between individuals according to a Gamma distribution with parameters m and k.

5) Customers' brand-choice probabilities and average-purchase-frequencies are distributed independently over the population.

From assumptions 1-5, it follows that: (i) the number of purchases of the product category made by all individuals in a certain time period follows a Negative-Binomial distribution with mean mT and exponent k; (ii) the number of purchases an individual makes of each of the g brands in a period of time T is given by the following expression, which Goodhardt, Ehrenberg and Chatfield (1984) called the NBD-Dirichlet model:

$$f_{k,m,\alpha_1,\dots,\alpha_g}(r_1,\dots,r_g) = f(r\mid m,k)\, f_{\alpha_1,\dots,\alpha_g}(r_1,\dots,r_g\mid r_1+\dots+r_g=r) =$$
$$\frac{(k+r-1)!}{r!(k-1)!}\left(\frac{k}{m+k}\right)^k\left(1-\frac{k}{m+k}\right)^r \frac{\Gamma(\alpha_1+\dots+\alpha_g)k!}{\Gamma\left(\sum_{j=1}^{g}\alpha_j+r\right)}\prod_{j=1}^{g}\frac{\Gamma(\alpha_j+r_j)}{r_j!\Gamma(\alpha_j)}$$

The above authors proposed an iterative method for model estimation which requires summary statistics as input values, such as brand penetrations b_j and average purchase rates m_j. The Dirichlet model has been used for many years. Originally, the calculations had to be done by hand, and later with DOS-based software (Uncles 1989); at the present time various tools are freely available on-line, for example, a software developed as an Excel Workbook by Kearns (2002), with a User's Guide written by Bound (2009a). Another estimation

procedure freely available is that composed using the programming language R by Chen (2008).
The iterative estimation method proposed by the authors of the model needs very simple input data: penetration and average purchase rates for the category and the various brands. When the above data are supplied, the method produces a series of brand performance measures, both for the time period of the data supplied and for other time periods, such as penetration, the percentage of customers buying the brand once or more times, average number of purchases of the brand and of the category per buyer of that brand, measures of loyalty, and measures of duplication, i.e., the proportion of customers of a brand who also buy a specific other brand in the period.

To activate the model, $g+2$ quantities need to be estimated: m, k, $\alpha_1,\ldots,\alpha_g$. With the g observed per capita purchase rates m_j, the iterative estimation procedure calculates the category purchase rate as $m = \sum_{j=1}^{g} m_j$ and equates the theoretical and observed market shares:

$$\frac{\alpha_j}{\sum_{j=1}^{g} \alpha_j} = \frac{m_j}{\sum_{j=1}^{g} m_j},$$

as the brand market shares must add to 1, there are $g-1$ equations to be solved.

Parameter k is calculated by fitting an NBD model to the distribution of purchases of the product category.

Both software types considered in this paper (one based on the Excel Workbook and the other in R), estimate the parameters starting with the method of Goodhardt, Ehrenberg and Chatfield (1984) but offering some different options that will be described in detail later in the paper.

Rungie (2003a) describes the use of likelihood theory to estimate the parameters of the Dirichlet model, providing an alternative to the standard procedure based on the method of zeros and ones and on marginal moments (Rungie 2003b). The likelihood approach to estimation is more efficient and is well suited to the extensions of the Dirichlet model, e.g., its development into a generalized model, with the inclusion of covariates such as marketing mix variables and consumers' characteristics (Rungie & Goodhardt 2004). In order to write the likelihood function, the data should be in the form of joint frequencies, like those contained in a contingency table with n rows, representing the number of consumers, and g columns, for the number of brands.

Alternatively, the iterative procedures based on the approach proposed by Goodhardt, Ehrenberg and Chatfield (1984) are computationally easy to use, quick, and require only aggregated data as input, as access to original panel data is not necessary. Raw panel data cannot always be used since panel operators who measure sales and household consumption provide information only in some aggregate format such as market share, penetration, and average purchase rate with reference to the various brands (Wright et al. 2002). In these situations, the only way to estimate the Dirichlet model is to use the traditional method. Dirichlet modelling continues to be a successful and influential approach, and is increasingly

being used to provide norms against which brand performance can be interpreted (see, among others, Uncles et al. 1995; Bhattacharya 1997; Ehrenberg et al. 2000).
From the viewpoint of practical applications, the Dirichlet model is useful for various objectives. Estimated values can be used to provide norms for stationary markets, to supply baselines for interpreting change (i.e., non-stationary situations) without having to match the results against a control sample, to help strategic decision-making, and to understand the nature of markets.

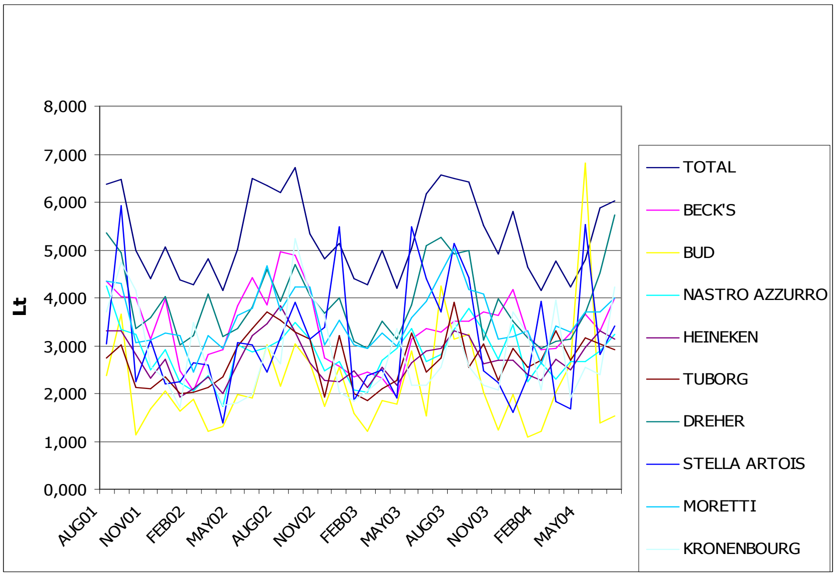
For the above reasons, it would be interesting to compare estimation results obtained by applying the various available software to perform iterative estimation.

The Data and the Italian Beer Market

The data used here refer to monthly purchases of 9 brands of beer (Moretti, Heineken, Nastro Azzurro, Dreher, Tuborg, Beck's, Stella Artois, Bud, Kronembourg) by Italian families in the period from August 2001 to July 2004. For each month, data is available for the number of families buying each brand, product category, brand market shares, brand and product average purchase rate, and average purchase frequency.

Figure 1 shows average purchases of beer in litres for the 9 brands and the product category. The market shows a clear seasonal pattern, with consumption increasing in summer.

Figure 1. Average purchases per household in lt, August 2001–July 2004



In the last 15 years, Italy's beer market has shown interesting changes in both supply and demand. Total consumption has increased, although average per capita consumption is still substantially lower than in many other European countries such as Greece, Spain and, of course, Germany. Consumption is also linked to warm weather, unlike the situation in Northern Europe, where consumption is distributed throughout the year. Production is concentrated, with a few large groups producing over three-quarters of the total product. Instead, the market is characterized by a quite high number of competing brands. In this paper, the 9 most popular brands are examined and Table 1 lists their market shares over the study period. Due to the nature of the available data, year 1 goes from August 2001 to July 2002, year 2 from August 2002 to July 2003, and year 3 from August 2003 to July 2004.

Table 1. Market shares, in percentages, per brand, August 2001-July 2004

	Year 1	Year 2	Year 3
Moretti	13.21	13.31	14.48
Dreher	8.95	8.59	8.15
Heineken	8.09	7.64	8.43
Beck's	3.72	4.04	4.34
Tuborg	3.26	3.36	3.05
Nastro Azzurro	4.00	3.87	1.16
Kronembourg	1.45	1.41	1.05
Stella Artois	1.24	1.00	0.72
Bud	1.13	1.14	0.81

Table 2 compares market evolution for the 9 brands in the three-year analysis. Consumption of the product category increased in the period, whereas consumption of our group of brands decreased. Nastro Azzurro, Stella Artois, Bud and Kronembourg contributed to this negative result, whereas Moretti is the brand which most increased its consumption. Regarding the number of families buying the product in the study period our set of brands performed better than the category; Moretti, again, showed the greatest increase.

Table 2. Market evolution for 9 brands in the three-year study period – in thousands

	Δ consumption in litres in thousands			Δ no. of families buying at least once in year in thousands		
	Year 1 to year 2	Year 2 to year 3	Year 1 to year 3	Year 1 to year 2	Year 2 to year 3	Year 1 to year 3
Moretti	4,885	7,796	12,681	274	810	1,084
Dreher	1,420	-406	1,014	-3	243	240
Heineken	1,609	4,105	5,714	133	-45	88
Beck's	2,633	2,124	4,758	217	389	606
Tuborg	1,514	-762	752,091	342	103	445
Nastro Azzurro	759	-11,626	-11	-151	260	109
Kronembourg	291	-1,344	-1,053	145	-85	60
Stella Artois	-648	-1,112	-1,760	-132	-38	-170
Bud	439	-1,288	-849	-84	-69	-153
Total	12,904	-2,513	10,391	741	1,568	2,249
Category	33,515	18,466	51,391	408	1,576	1,984

Results and Discussion

With data on beer consumption in Italy, the Dirichlet model was estimated with two types of software applying the method of moments. Both estimate parameters m, k and S. One software application, written in programming language R and called the Dirichlet Package (DP) (Chen 2008), requires as input data category (b) and brand (b_j) penetration, category

purchase frequency (w) and brand market shares. Specifically, the program uses observed category penetration and purchase frequency to estimate m and k and observed brand penetrations and market shares to estimate S.
The other procedure, based on an Excel Workbook (EW) (Kearns 2002 and 2009), requires as input data category (b) and brand (b_j) penetration, and average purchase frequency for category (w) and the various brands (w_j); if brand penetrations are not available, market shares can be used. In this application brand penetrations have been used as input for estimation.

Table 3 lists estimated parameters m, k and S for the three years with the two types of software and shows that estimated parameters for the NBD part of the model are the same, whereas differences occur in estimating S. This result may be due to outlier values for parameters α_j because of the presence in the market of atypical brands and to the differences in estimating the parameter S in the two procedures.

Clear evidence that Nastro Azzurro is an atypical brand in that its market share decreases from 4% in year 1 to 1% in year 3; its market its not stationary (see Table 1). Bound (2009a) suggested excluding such brands when estimating the overall value of S. The EW calculates a value of S separately for each brand so that the prediction of penetration for that brand is exact. These estimates are then combined and an overall value of S is applied to all data. DP estimates S directly from observed brand penetrations and market shares.

Table 3. Dirichlet model estimates with the two types of software

	Dirichlet Package	**Excel Workbook**
Year 1		
m	18.48	18.50
k	.36	.36
S	.69	.90
Year 2		
m	20.03	20.00
k	.38	.38
S	.76	.90
Year 3		
m	20.87	20.90
k	.50	.50
S	.70	1.60

Parameters k and S are characteristics of the product class and may be interpreted as reflecting consumers' heterogeneity. In this market, low k values indicate that purchase frequencies vary greatly among buyers, whereas high S values mean that purchase probabilities do not differ greatly among consumers of that brand[1].

Both software types make predictions of the market behaviour estimating some brand performance measures.

[1] S measures the diversity of the brand purchase propensity across consumers: high values imply less diversity (Bound, 2009).

The DP estimates category (b) and brand (b_j) penetration, average purchase frequency per brand (w_j), average purchase frequency per category per buyers of the brand (w_{Pj}), average number of purchases per brand and its distribution by buyers of the brand, brand penetration and average purchase frequency among category buyers with a specific frequency range and duplication measures.

The EW estimates category (b) and brand (b_j) penetration, average purchase frequency per brand (w_j), average purchase frequency per category per buyer of the brand (w_{Pj}), percentage buying the brand once and five or more times, percentage of sole buyers, rate of purchase of sole buyers, percentage of repeat buying from period to period and duplication measures.

Table 4. Penetration and frequency of purchase by brand: observed, and estimated with DP and with EW, for three-year study period, with brand Nastro Azzurro

	b_j								
	Year 1			**Year 2**			**Year 3**		
	Observed	**DP**	**EW**	**observed**	**DP**	**EW**	**observed**	**DP**	**EW**
Moretti	.22	.22	.24	.24	.24	.25	.27	.27	.36
Dreher	.15	.15	.17	.15	.16	.17	.16	.16	.23
Heineken	.13	.13	.15	.14	.14	.15	.14	.17	.23
Beck's	.06	.07	.07	.07	.08	.09	.09	.09	.13
Tuborg	.07	.06	.07	.09	.07	.07	.09	.07	.09
Nastro Azzurro	.11	.07	.07	.11	.08	.08	.12	.03	.04
Kronembourg	.04	.03	.03	.05	.03	.03	.04	.02	.03
Stella Artois	.04	.02	.04	.03	.02	.02	.03	.02	.02
Bud	.03	.02	.02	.03	.02	.03	.02	.02	.03
	w_j								
Moretti	10.94	11.27	10.39	11.31	10.74	10.74	11.08	11.18	8.41
Dreher	10.90	10.76	9.85	11.34	10.10	10.10	10.45	10.40	7.48
Heineken	11.47	10.65	9.74	11.20	9.97	9.97	13.08	10.43	7.52
Beck's	11.11	10.13	9.21	11.25	9.50	9.50	10.10	9.94	6.95
Tuborg	8.52	10.08	9.15	10.05	9.41	9.41	6.99	9.80	6.77
Nastro Azzurro	6.55	10.16	9.24	10.11	9.47	9.47	2.05	9.58	6.52
Kronembourg	6.81	9.87	8.94	9.79	9.16	9.16	5.22	9.57	6.51
Stella Artois	6.30	9.84	8.91	9.74	9.11	9.11	5.20	9.52	6.46
Bud	7.19	9.82	8.90	9.75	9.13	9.13	7.72	9.57	6.51

Table 4 lists some estimation results with reference to the market. The parameters estimated with the two types of software are compared with observed values. The results confirm that Nastro Azzurro is quite atypical in this market, especially in the third year of observation. Following Bound's (2009a) suggestion, the model was re-estimated excluding this brand (Table 5).

Table 5. Penetration and frequency of purchase by brand: observed, and estimated with DP and with EW, for three-year study period, without brand Nastro Azzurro

	b_j								
	Year 1			**Year 2**			**Year 3**		
	observed	**DP**	**EW**	**observed**	**DP**	**EW**	**observed**	**DP**	**EW**
Moretti	.22	.22	.22	.24	.23	.24	.27	.27	.28
Dreher	.15	.15	.16	.15	.16	.17	.16	.16	.17
Heineken	.13	.14	.15	.14	.14	.15	.14	.17	.18
Beck's	.06	.07	.07	.07	.08	.08	.09	.09	.10
Tuborg	.07	.06	.06	.09	.07	.07	.09	.07	.07
Kronenbourg	.04	.03	.03	.05	.03	.03	.04	.02	.02
Stella Artois	.04	.02	.02	.03	.02	.02	.03	.02	.02
Bud	.03	.02	.02	.03	.02	.02	.02	.02	.02
	w_j								
Moretti	10.94	11.27	10.85	11.31	11.34	11.05	11.08	11.18	10.81
Dreher	10.90	10.76	10.32	11.34	10.72	10.42	10.45	10.40	10.02
Heineken	11.47	10.65	10.22	11.20	10.59	10.29	13.08	10.43	10.05
Beck's	11.11	10.13	9.70	11.25	10.12	9.83	10.10	9.94	9.56
Tuborg	8.52	10.08	9.64	10.05	10.05	9.74	6.99	9.80	9.41
Kronembourg	6.81	9.87	9.43	9.79	9.79	9.49	5.22	9.57	9.47
Stella Artois	6.30	9.84	9.41	9.74	9.74	9.44	5.20	9.52	9.14
Bud	7.19	9.82	9.40	9.75	9.75	9.46	7.72	9.53	9.47

In order to compare the two estimation procedures, the Mean Average Percentage Error (MAPE) was calculated on results reported in Tables 4 and 5. Figures in Table 6 show that both procedures are not robust in the presence of atypical brands in the market, since excluding Nastro Azzurro increases model fit. Error reduction is greater in the estimation of average purchase frequency but it is noticeable also in the estimation of brand penetration. As already pointed out by Bound (2009), the EW procedure has a worse fit in the estimation of the b_j parameter, but, in this application, it shows a better fit in the estimation of brand purchase frequencies.

Table 6. Mean Average Percentage Error (MAPE) for *b* and *w* with and without brand Nastro Azzuro, DP and EW software

	b		***w***	
	DP	EW	DP	EW
with Nastro Azzurro	21.20	26.75	33.86	26.22
without Nastro Azzurro	15.85	22.71	17.03	15.13
Δ	-5.35	-4.04	-16.83	-11.09

Table 7 contains some other brand performance measures that help deeper analysis of the Italian beer market obtained with the EW software. This deeper analysis is provided both for the insights it may offer in the current project, and also to facilitate any future secondary research that uses the data presented in this paper.

Table 7. Percentage of consumers 100% loyal, percentage of consumers who repeat purchase in the period and category average purchase frequency by buyers of the brand, for three-year study period, without brand Nastro Azzurro, EW

	Year 1			Year 2			Year 3		
	100% loyal	**Repeat buying %**	w_{Pj}	**100% loyal**	**Repeat buying %**	w_{Pj}	**100% loyal**	**Repeat buying %**	w_{Pj}
Moretti	13.32	84.04	13.40	11.86	84.45	13.48	12.05	85.49	14.29
Dreher	12.16	83.37	13.51	10.65	83.68	13.60	10.41	84.42	14.44
Heineken	11.93	83.22	13.54	10.43	83.52	13.53	10.58	84.47	14.44
Beck's	10.86	82.51	13.65	9.60	82.91	13.72	9.54	83.75	14.54
Tuborg	10.73	82.43	13.66	9.45	82.79	13.73	9.25	83.52	14.57
Kronembourg	10.37	82.12	13.71	9.04	83.45	13.78	8.83	83.16	14.62
Stella Artois	10.30	82.09	13.71	8.96	82.38	13.79	8.77	83.10	14.63
Bud	10.27	82.07	13.71	8.99	82.41	13.79	8.79	83.12	14.63

The first observation emerging from Tables 5 and 7 is that the Italian beer market is segmented. It is possible to identify two groups of brands with similar behaviour within the group and different behaviour between the groups. The first segment is composed of brands Tuborg, Kronembourg, Stella Artois and Bud, the smaller brands. These show estimated penetrations lower than observed ones and estimated average purchase frequencies of the brand higher than observed ones and the lowest percentages of loyal customers. It is noticeable that in this market the common double jeopardy effect is not present. The second segment is composed of brands Moretti, Dreher, Heineken and Beck's. They show estimated penetrations equal to or higher than observed ones, estimated purchase frequencies for the brand lower than observed ones and the highest percentages of loyal customers. As it appears from the figures listed in Table 7, differences in percentages of repeat buying and purchase frequency of the category are not particularly big in magnitude, nevertheless they exhibit a clear trend. How often customers buy the whole category increases slightly with decreasing penetration (this is the natural monopoly effect identified by McPhee, 1963). Percentages of repeat buying slightly decrease with brand share.

The first group of brands shows low market shares (the four brands cover together less than 7% of the market), low loyalty but heaviest buyers for the category, it may be defined as a niche segment. The second group shows higher market shares, the highest percentages of loyal customers, the highest percentages of repeat buying, it can be defined a mass market segment with many light buyers.

Conclusions

Application of the Dirichlet model to the Italian beer market shows that it is segmented into at least two parts, massive consumption one hand and a niche, in which consumers behave quite differently. Moreover, an atypical brand is present with a market that seems not to be stationary in the period considered. Many applications of the Dirichlet model have shown that, even when the market is not quite steady, or when some clustering occurs, the model mostly still holds and it provides useful benchmarks (see, for example, Ehrenberg et al.

2004). This paper shows again how the model can be used to assess how existing brands are performing.

The model is very parsimonious, at least when the method of moments is used for parameters estimation. In this case only a few numerical inputs are needed, typically penetrations and average purchase frequencies of the category and the various brands. In this paper the Dirichlet model is estimated with two software packages, an Excel based one and another written with programming language R. Both procedures rely on the iterative estimation method proposed by Goodhardt, Ehrenberg and Chatfield (1982), but they differ in some aspects relating input, output and estimation algorithms.

Results obtained with two available types of software for the methods of moments are compared here. The software based on Excel Workbook turns out to be less precise in the estimation of brand penetration while more precise in the estimation of purchase frequencies. Neither procedure was very robust in the presence of atypical brands in the market. Lack of robustness does not affect estimation of the parameters of the NBD component of the model but, as it does affect all other parameters, it is advisable to eliminate such brands when conducting analysis, for reliable results. The evidence presented above suggests that it is possible to use either software package to estimate the Dirichlet model in order to analyze and possibly forecast consumers' behaviour in a competitive market.

It would be interesting at this point to compare parameters estimated by maximizing the loglikelihood function with those presented here. However, this exercise would require raw panel data which are currently not available.

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